Published by Institute of Physics Publishing for SISSA

Received: *October 31, 2005*
Revised: *December 15, 2005*
Accepted: *December 30, 2005*
Published: *January 12, 2006*

Transverse momentum dependent factorization for radiative leptonic decay of B-meson

Jian-ping Ma and Qi Wang

Institute of Theoretical Physics, Academia Sinica
Beijing 100080, China
E-mail: majp@itp.ac.cn, wangqi@itp.ac.cn

Abstract: With a consistent definition of transverse-momentum-dependent(TMD) light-cone wave function of B-meson, we show that the amplitude of the radiative leptonic B-decay can be factorized at one-loop level as a convolution with the wave function and a perturbative coefficient function, combined with a soft factor. In this TMD factorization, the transverse momenta of partons in the B-meson are taken into account and all soft divergences are contained in the wave function and the soft factor. The coefficient function is infrared-safe. The factorization works on a diagram-by-diagram basis and is possible to extend beyond one loop. With the factorization the large logarithms in the perturbative function can be simply resummed. Our work shows that the result of collinear factorization for the decay can be derived from that of TMD factorization. Therefore, the two factorizations for the case here are simply related to each other.

Keywords: QCD, B-Physics.

JHEP01(2006)067

Contents

1. Introduction

Exclusive B-decays play an important role for testing the standard model and seeking for new physics. Experimentally they are studied intensively. Theoretically, there are two approaches of QCD factorization for studying these decays. One is based on the collinear factorization [1], in which the transverse momenta of partons in a B-meson are integrated out and their effect at leading twist is neglected. The collinear factorization has been proposed for other exclusive processes for long time [2]. Another one is based on k_T-factorization [3] or pQCD approach, where one takes the transverse momenta k_T of partons into account at leading twist by means of k_T-dependent light-cone wave function. We will call such a factorization as transverse momentum dependent(TMD) factorization. The advantage of the TMD factorization is that it may eliminate end-point singularities in collinear factorization [4] and some higher-twist effects are included. The knowledge of the transverse momentum dependent(TMD) light-cone wave function will provide a 3-dimensional picture of a B-meson bound state. However, it was not clear how to define the TMD light-cone wave function in a consistent way to perform a TMD factorization because of light-cone singularities [5].

Similar problems also appear in defining TMD parton distributions and fragmentation functions if one tries to do TMD factorization for inclusive processes. In general the light-cone singularities appear if a parton emits gluons carrying momenta which are vanishingly small in the +-direction but large in other directions in a light-cone coordinate system. In a collinear factorization for an exclusive or inclusive process, these singularities are cancelled between different contributions if the transverse momentum of the parton is integrated out. If the transverse momentum is not integrated, the singularities are not cancelled.

For inclusive processes like Drell-Yan, semi-inclusive DIS etc., it has been shown that one can consistently define TMD parton distributions by using gauge links in the direction off the light-cone direction and the TMD factorization of inclusive processes can be done without light-cone singularities [6 − 9]. The TMD parton distributions defined with these gauge links will depend on the deviation of the direction from the light-cone direction. The evolution with this dependence is controlled by the Collins-Soper equation [6] which leads to the so-called CSS resummation formalism [7 − 9]. This formalism is for resummation of large logarithms appearing in the collinear factorization. In that sense the TMD- and collinear factorization are related to each other. But the similar relation in exclusive B-decays has not been studied.

We have proposed in [10] to consistently define the TMD light-cone wave function of B-meson by using gauge links off the light-cone direction and studied its relation to the usual light-cone wave function in the collinear factorization. With the consistent definition it is important to show that the TMD factorization can be consistently performed. The relation between two factorization approaches may be then established. As a first step towards these goals we study in this paper TMD factorization for the radiative leptonic decay of B-meson. We also plan to study TMD factorization for $B \to \pi$ form factor and other decay processes. It should be noted that the definition of the TMD light-cone wave function is not unique, different definitions are possible. A different definition can be found in [11]. With different definitions the most important thing is to show that one can perform TMD factorization consistently with one of these definitions, at least at one-loop level. To our knowledge, there is so far no such a study beyond tree-level for exclusive B-decays. We will show that with the definition given in [10] the factorization can be done at one-loop level for the process studied in this paper.

The radiative leptonic B-decay has been studied extensively [12 − 16, 18]. The effect of strong interaction in the decay is parameterized with form factors. These form factors have been studied in [13 − 18] with QCD factorization. It has been shown that the form factors can be factorized as a convolution with a perturbative coefficient function and the light-cone wave function of the B-meson in the collinear factorization [14 − 16, 18]. In these works the transverse momentum of partons is integrated out. It results in the convolution only with the +-component of the parton momentum. In [13] the transverse momentum of the parton is not integrated and is explicitly taken into account, but the consistency of the definition of the TMD light-cone wave function is not addressed and the problem of the gauge invariance of the definition is ignored. In [17] the decay is studied with k_T-factorization or TMD factorization, but the TMD light-cone wave function employed there has the light-cone singularity.

With our gauge-invariant definition we can show with TMD factorization that the form factors take the factorized form:

$$\phi_+ \otimes \tilde{S} \otimes H. \tag{1.1}$$

In the above ϕ_+ is the TMD light-cone wave function, $\tilde{S}$ is a soft factor, H is a coefficient function which can be calculated with perturbative QCD and is free from soft divergences. ϕ_+ and $\tilde{S}$ are well-defined matrix elements of QCD operators. The convolution here is not only with +-components but also transverse components of parton momenta. In this paper we prove the factorization at one-loop level. We show that the cancellation of all soft divergences is on a diagram-by-diagram basis. This is important for extending our factorization beyond one-loop level. In the case studied in the paper, the TMD factorization is similar to the collinear factorization because there is no parton or hadron in the final state. But it is important to show first that the TMD factorization works for this simple case and then extend the TMD factorization to other cases. An interesting fact with the TMD factorization is that it provides a simple way to resum large logarithms in the perturbative function H, as we will show in the paper.

As mentioned before, TMD factorization for an inclusive process can be related to the corresponding collinear factorization. One can expect that such a relation also exists for exclusive B-decays. Indeed, in the case studied here, such a relation exists and it is simple: Both factorizations are equivalent, i.e., one can derive the result of the collinear factorization from our TMD factorization. We will show this in this work. One reason for this simple relation is that there is no hadron, hence any parton in the final state.

Our paper is organized as the following: In the next section we define the TMD light-cone wave function and give its one-loop result in detail with a general partonic state, which will be used to perform TMD factorization. A detailed discussion about the TMD light-cone wave function and its relation to the usual light-cone wave function in collinear factorization can be found in [10]. In section 3 we introduce our notation for the decay and the result of the factorization at tree-level. In section 4 we will complete the factorization at one-loop level and determine the soft factor. In section 5 we show that the result of the collinear factorization can be derived from that of the TMD factorization and establish the relation between the two factorizations for the decay. In section 6 we will make an attempt to re-sum large logarithms in TMD factorization. Section 7 is our conclusion and outlook.

2. A consistent definition of the TMD light-cone wave-function

In this section we give our definition of the TMD light-cone wave function and its one-loop result in detail with a general partonic state. A brief report of the result and the study of the relation to the light-cone wave function in the collinear factorization can be found in [10].

We will use the light-cone coordinate system, in which a vector a^μ is expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$ and $a_\perp^2 = (a^1)^2 + (a^2)^2$. For b-quark we will use the heavy quark effective theory(HQET). To define the TMD light-cone wave function we introduce a vector $u^\mu = (u^+, u^-, 0, 0)$ and the definition is given in the limit $u^+ \ll u^-$ [10]:

$$\phi_+(k^+, k_\perp, \zeta, \mu) = \int \frac{dz^-}{2\pi} \frac{d^2 z_\perp}{(2\pi)^2} e^{ik^+ z^- - i\vec{z}_\perp \cdot \vec{k}_\perp} \times \langle 0|\bar{q}(z) L_u^\dagger(\infty, z)\gamma^+\gamma_5 L_u(\infty, 0) h(0)|\bar{B}(v)\rangle|_{z^+=0}, \tag{2.1}$$

where $h(x)$ is the b-quark field in HQET. and L_u is the gauge link in the direction u:

$$L_u(\infty, z) = P\exp\left(-ig_s\int_0^\infty d\lambda u\cdot G(\lambda u + z)\right). \tag{2.2}$$

In the above, the B-meson moves with the velocity $v^\mu = (v^+, v^-, 0, 0)$, i.e., in the z-direction. The limit should be understood that we do not take the contributions proportional to any positive power of u^+/u^- into account. This definition is gauge invariant in any non-singular gauge in which the gauge field is zero at infinite space-time. It has not the mentioned light-cone singularity as we will show through our one-loop result, but it has an extra dependence on the momentum k^+ through the variable $\zeta^2 = 4(u\cdot k)^2/u^2 = \zeta_u^2(k^+)^2$, or an extra dependence on ζ_u^2. The evolution with the renormalization scale μ is simple:

$$\mu\frac{\partial\phi_+(k^+, k_\perp, \zeta, \mu)}{\partial\mu} = (\gamma_q + \gamma_Q)\phi_+(k^+, k_\perp, \zeta, \mu), \tag{2.3}$$

where γ_q and γ_Q is the anomalous dimension of the light quark field q and the heavy quark field h in the axial gauge $u\cdot G = 0$, respectively. In the remainder of the paper we will not indicate the μ-dependence explicitly if it does not cause any confusion. It should be noted that one can not simply relate $\phi_+(k^+, k_\perp, \zeta)$ by integrating $k_\perp$ to the light-cone wave function in the collinear factorization, whose definition can be found in [19]. The reason for this has been discussed in detail in [10].

To perform TMD factorization one needs to calculate the wave function with perturbative QCD, in which the B-meson is replaced by a partonic state. We take the partonic state $|b(m_b v + k_b), \bar{q}(k_q)\rangle$ to replace the B-meson in the definition, the momenta are given as

$$k_q^\mu = (k_q^+, k_q^-, \vec{k}_{q\perp}), \qquad k_b^\mu = (k_b^+, k_b^-, -\vec{k}_{q\perp}). \tag{2.4}$$

These partons are on-shell, i.e., $k_q^2 = m_q^2$ and $v\cdot k_b = 0$ in HQET. It should be noted that we take a finite $k_{q\perp}$ without loosing generality. The quark mass m_q will regularize collinear singularities. We also introduce a gluon mass λ to regularize infrared singularities. The variable k^+ of the wave function is from 0 to ∞ in the heavy quark limit. Actually, from the momentum conservation, it is from 0 to $P^+ = m_b v^+ + k_b^+ + k_q^+$. Under the limit $m_b \to \infty$ we have $P^+ \to \infty$. As discussed in [10], if we set P^+ to be ∞ at the beginning, it may result in some ill-defined distributions. Therefore we should take a finite P^+ in the calculation and take the limit $P^+ \to \infty$ in the final result. For results obtained in this paper we will take the limit where it does not introduce any problem.

At tree-level, the wave function reads:

$$\phi_+^{(0)}(k^+, k_\perp, \zeta) = \bar{v}(k_q)\gamma^+\gamma_5 u(k_b)\delta(k^+ - k_q^+)\delta^2(\vec{k}_\perp - \vec{k}_{q\perp}). \tag{2.5}$$

We will always write a quantity A as $A = A^{(0)} + A^{(1)} + \cdots$, where $A^{(0)}$ and $A^{(1)}$ stand for tree-level- and one-loop contribution respectively. At one-loop one can divide the corrections into a real part and a virtual part. The real part comes from contributions of Feynman diagrams given in figure 1. The virtual part comes from contributions of Feynman diagrams given in figure 2, these contributions are proportional to the tree-level result.

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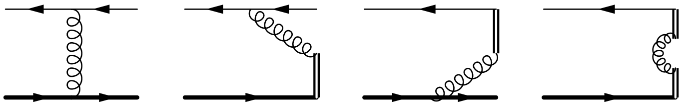

Figure 1: Diagrams of one-loop contributions. Thick lines stand for b-quark, double lines represent gauge links.

To illustrate how to calculate these contributions and how the limit $u^+ \ll u^-$ is taken, let us consider the contribution from figure 1c. After integrating the $-$-component of the momentum carried by the exchanged gluon the contribution reads:

$$\begin{aligned}\phi_+(k^+, k_\perp, \zeta)|_{1c} &= -\frac{2\alpha_s}{3\pi^2}\bar{v}(k_q)\gamma^+\gamma_5 u(k_b) u\cdot v\theta(k^+ - k_q^+)\\ &\cdot \frac{2q^+}{v^+(q_\perp^2+\lambda^2)+2v^-(q^+)^2+i0}\cdot\frac{1}{u^+(q_\perp^2+\lambda^2)+2u^-(q^+)^2+i0}, \end{aligned} \tag{2.6}$$

with $q^+ = k_q^+ - k^+$ and $\vec{q}_\perp = k_{q\perp} - k_\perp$. If we simply set $u^+ = 0$, the contribution is proportional to $1/(k^+ - k_q^+)$ and divergent at $k^+ = k_q^+$. This is the mentioned light-cone singularity. With the nonzero u^+ the contribution is finite for any k^+. The contribution in the limit $u^+ \ll u^-$ can be derived by taking the contribution as a distribution of k^+ and it reads:

$$\begin{aligned}\phi_+(k^+, k_\perp, \zeta)|_{1c} &= -\frac{2\alpha_s}{3\pi^2}\bar{v}(k_q)\gamma^+\gamma_5 u(k_b)\\ &\cdot\left[\left(\frac{\theta(k^+ - k_q^+)}{q^+(q_\perp^2+\lambda^2+\zeta_v^2(q^+)^2)}\right)_+ - \delta(k^+ - k_q^+)\frac{1}{2(q_\perp^2+\lambda^2)}\ln\frac{\zeta_u^2}{\zeta_v^2}\right] + \mathcal{O}(\zeta_u^{-2}),\\ \zeta_u^2 &= \frac{2u^-}{u^+} = \frac{\zeta^2}{(k^+)^2}, \qquad \zeta_v^2 = \frac{2v^-}{v^+}. \end{aligned} \tag{2.7}$$

In the above the limit $P^+ \to \infty$ is already taken. From the result we can see that the light-cone singularity is regularized with the finite but large ζ_u^2. The other contributions of the real part are:

$$\begin{aligned}\phi_+(k^+, k_\perp, \zeta)|_{1a} &= \frac{2\alpha_s}{3\pi^2}\bar{v}(k_q)\gamma\cdot v(\gamma\cdot(q-k_q)+m_q)\gamma^+\gamma_5 u(k_b)F_1\\ F_1 &= -i\int\frac{dq^-}{2\pi}\cdot\frac{1}{(q-k_q)^2-m_q^2+i0}\cdot\frac{1}{q^2-\lambda^2+i0}\cdot\frac{1}{v\cdot q+i0},\\ \phi_+(k^+, k_\perp, \zeta)|_{1b} &= \frac{2\alpha_s}{3\pi^2}\bar{v}(k_q)\gamma^+\gamma_5 u(k_b)\\ &\cdot\left[\frac{k^+}{\Delta_q}\left(\frac{\theta(k_q^+ - k^+)}{k_q^+ - k^+}\right)_+ + \frac{1}{2(q_\perp^2+\lambda^2)}\delta(k^+ - k_q^+)\ln\frac{\zeta^2}{q_\perp^2+\lambda^2}\right],\\ \Delta_q &= k_q^+((q_\perp - xk_{q\perp})^2 + x^2 m_q^2 + (1-x)\lambda^2), \quad x = 1-\frac{k^+}{k_q^+},\\ \phi_+(k^+, k_\perp, \zeta)|_{1d} &= -\frac{2\alpha_s}{3\pi^2}\bar{v}(k_q)\gamma^+\gamma_5 u(k_b)\delta(k^+ - k_q^+)\frac{1}{q_\perp^2+\lambda^2}. \end{aligned} \tag{2.8}$$

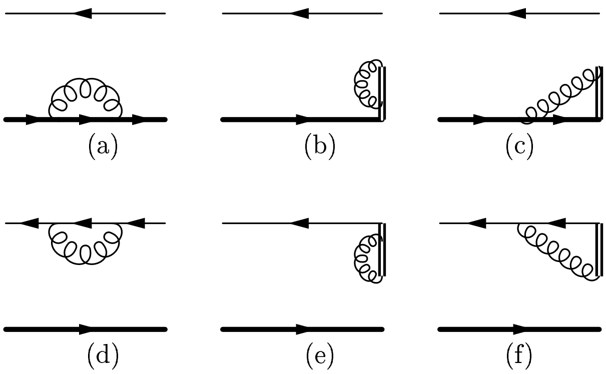

Figure 2: The virtual part of the one-loop correction.

The integral F_1 for the contribution from figure 1a can be done easily, but it results in a lengthy expression. We will show later that the contribution will not affect the perturbative coefficient function H.

The virtual part of the one-loop correction is from the Feynman diagrams given in figure 2. Contributions from each diagrams are:

$$\begin{aligned}
\phi_+(k^+,k_\perp,\zeta)|_{2a} &= \phi_+(k^+,k_\perp,\zeta)|_{2b} = \phi_+(k^+,k_\perp,\zeta)|_{2e} = \phi_+^{(0)}(k^+,k_\perp,\zeta)\cdot\frac{\alpha_s}{3\pi}\ln\frac{\mu^2}{\lambda^2},\\
\phi_+(k^+,k_\perp,\zeta)|_{2c} &= -\phi_+^{(0)}(k^+,k_\perp,\zeta)\cdot\frac{\alpha_s}{3\pi}\ln\frac{\mu^2}{\lambda^2}\ln\frac{\zeta_u^2}{\zeta_v^2},\\
\phi_+(k^+,k_\perp,\zeta)|_{2f} &= \phi_+^{(0)}(k^+,k_\perp,\zeta)\cdot\frac{\alpha_s}{6\pi}\left[2\ln\frac{\mu^2}{m_q^2}+2\ln\frac{\zeta^2}{m_q^2}-\ln^2\frac{\zeta^2}{m_q^2}\right.\\
&\qquad\left.-2\ln\frac{m_q^2}{\lambda^2}\ln\frac{\zeta^2}{m_q^2}-\frac{2\pi^2}{3}+4\right],\\
\phi_+(k^+,k_\perp,\zeta)|_{2d} &= \phi_+^{(0)}(k^+,k_\perp,\zeta)\cdot\frac{\alpha_s}{6\pi}\left[-\ln\frac{\mu^2}{m_q^2}+2\ln\frac{m_q^2}{\lambda^2}-4\right],
\end{aligned}\tag{2.9}$$

The complete one-loop contribution $\phi_+^{(1)}$ is the sum of contributions from the 10 Feynman diagrams in figure 1 and figure 2. With these results one can derive the evolution of ζ. For this we transform the wave-function into the impact parameter b-space:

$$\phi_+(k^+,b,\zeta,\mu) = \int d^2k_\perp e^{i\vec{k}_\perp\cdot\vec{b}}\phi_+(k^+,k_\perp,\zeta,\mu),\tag{2.10}$$

the evolution reads:

$$\zeta\frac{\partial}{\partial\zeta}\phi_+(k^+,b,\zeta,\mu) = \left[-\frac{4\alpha_s}{3\pi}\ln\frac{\zeta^2b^2e^{2\gamma-1}}{4}-\frac{2\alpha_s}{3\pi}\ln\frac{\mu^2e}{\zeta^2}\right]\phi_+(k^+,b,\zeta,\mu).\tag{2.11}$$

The first factor is the famous factor $K+G$ [6, 7], the last factor comes because we used HQET for the heavy quark.

Before ending the section, we briefly discuss the heavy quark limit $P^+\to\infty$. For the usual light-cone wave function, this limit will result in that the wave function is not normalizable as found in [19, 20] and it is shown through an explicit calculation with perturbative theory in [10]. For the TMD light-cone wave function it is normalizable if the transverse momentum is not integrated out. When we transform the TMD light-cone wave function into b-space, we should keep P^+ finite.

3. Notations and factorization at tree level

We consider the radiative decay of the B-meson $\bar{B}$ which contains at least a b-quark and a light anti-quark $\bar{q}$:

$$\bar{B} \to \gamma + \ell + \bar{\nu}. \tag{3.1}$$

We take a frame in which $\bar{B}$ moves in the z-direction with the velocity $v^\mu = (v^+, v^-, 0, 0)$ and the photon with the momentum $p^\mu = (0, p^-, 0, 0)$. It is worth to mention here that this decay has not been observed so far. An upper bound for the branching ratio is given in [21]:

$$\mathrm{Br}(\bar{B} \to \gamma + \ell + \bar{\nu}) < 2.0 \times 10^{-6}. \tag{3.2}$$

In the decay the effect of the strong interaction is controlled by a matrix element of the operator $\bar{q}\gamma^\mu(1-\gamma_5)b$ with $b(x)$ being the b-quark field in the full QCD. Since we use HQET for the heavy b-quark, the matrix element can be matched to HQET:

$$\langle \gamma(\epsilon^*, p)|\bar{q}\gamma^\mu(1-\gamma_5)b|\bar{B}(v)\rangle = f(\mu)\langle \gamma(\epsilon^*, p)|\bar{q}\gamma^\mu(1-\gamma_5)h|\bar{B}(v)\rangle, \tag{3.3}$$

where $f(\mu)$ is the matching coefficient. It is given by:

$$f(\mu) = 1 + \frac{\alpha_s(\mu)}{3\pi}\left(3\ln\frac{m_b}{\mu} - 2\right) + \mathcal{O}(\alpha_s^2). \tag{3.4}$$

The HQET matrix element can be parameterized as

$$\begin{aligned} \mathcal{T}^\mu &= \frac{1}{\sqrt{4\pi\alpha}}\langle \gamma(\epsilon^*, p)|\bar{q}\gamma^\mu(1-\gamma_5)h|\bar{B}(v)\rangle \\ &= \varepsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu v_\rho p_\sigma F_V(v\cdot p) + i\left(v\cdot p\epsilon^*_\mu - v\cdot\epsilon^* p^\mu\right)F_A(v\cdot p). \end{aligned} \tag{3.5}$$

In the rest frame of $\bar{B}$, $v \cdot p$ is the energy of the photon. The invariant $v \cdot p$ can be from 0 to $M_B/2$. The photon is emitted by quarks inside the B-meson. If $v \cdot p$ is large, i.e., $v \cdot p \gg \Lambda_{QCD}$ those quarks will change their momenta significantly, i.e., the emission becomes a short-distance process. This leads to that those form factors, hence the matrix element can be studied with perturbative QCD, in which one can separate short-distance- and long distance effect by factorization.

To show the factorization, one usually replaces hadronic states with reasonable parton states, then calculate processes which need to be factorized and nonperturbative objects like wave functions in our case to extract the perturbative coefficient functions. A factorization means at least that those coefficient functions do not contain any soft divergence. For our purpose we replace the $\bar{B}$ state $|\bar{B}\rangle$ with the partonic state $|\bar{q}b\rangle$. The momenta of the partons are the same as in eq. (2.4). At tree-level the contribution to the matrix element is given by the two diagrams in figure 3. The second diagram will not contribute in the

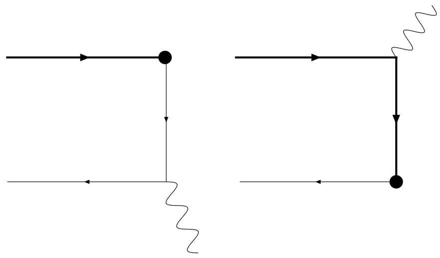

Figure 3: Tree-level contribution to the matrix element. The thick line is for the b-quark, the black dot denotes the insertion of the operator.

heavy quark limit by noting the fact $v \cdot \epsilon^* = 0$ for a real photon. The tree-level amplitude T^μ reads:

$$T_\mu^{(0)} = Q_q \bar{v}(k_q)\gamma \cdot \epsilon^* \cdot \frac{\gamma \cdot (p - k_q) + m_q}{(p-k_q)^2 - m_q^2}\gamma_\mu (1-\gamma_5)u(k_b), \tag{3.6}$$

where Q_q is the charge fraction of $\bar{q}$. In TMD factorization one will neglect the transverse momentum of initial partons in nominators of propagators but keep it in the denominators. The case studied here is rather special because the denominator does not depend on the transverse momentum. With a little algebra one can show that

$$\begin{aligned} T_\mu^{(0)} &= -\frac{Q_q}{2p \cdot k_q}\bar{v}(k_q)\gamma \cdot \epsilon^* \gamma \cdot p \gamma_\mu (1-\gamma_5)u(k_b) + \cdots \\ &= \frac{iQ_q}{2v \cdot p}\left[\varepsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}v^\rho p^\sigma + i\left(v \cdot p \epsilon_\mu^* - v \cdot \epsilon^* p_\mu\right)\right] \cdot \frac{1}{k_q^+}\bar{v}(k_q)\gamma^+ \gamma_5 u(k_b) + \cdots, \end{aligned} \tag{3.7}$$

where $\cdots$ denotes the neglected $k_{q\perp}$-dependence from the quark propagator and the contribution from the partonic state which does not have the same quantum numbers as $\bar{B}$ does.

With the tree-level result of the TMD light-cone wave function, we obtain the factorization for those form factors:

$$F_V = F_A = \frac{iQ_q}{2v \cdot p}\int dk^+ d^2k_\perp \phi_+(k^+, k_\perp, \zeta, \mu)\frac{1}{k^+}. \tag{3.8}$$

At the orders considered in the work, F_V is always the same as F_A. We will write our factorization formulas as:

$$F_V = F_A = \frac{iQ_q}{2v \cdot p}\int dk^+ d^2k_\perp dl^+ d^2l_\perp \phi_+(k^+, k_\perp, \zeta)\tilde{S}(l^+, l_\perp, \zeta_u)\theta(k^+ + l^+)H(k^+ + l^+, \zeta_u), \tag{3.9}$$

so that at the leading order of α_s the perturbative coefficient function H and the soft factor in perturbation theory at tree-level reads:

$$H^{(0)}(k^+, \zeta_u) = \frac{1}{k^+}, \qquad \tilde{S}^{(0)}(k^+, k_\perp, \zeta_u) = \delta(k^+)\delta^2(\vec{k}_\perp). \tag{3.10}$$

It is noted that at the leading order H does not depend on $k_\perp$ in the case studied here, while in the other cases like $B \to \pi$ transition it does. If one replaces the B-meson state

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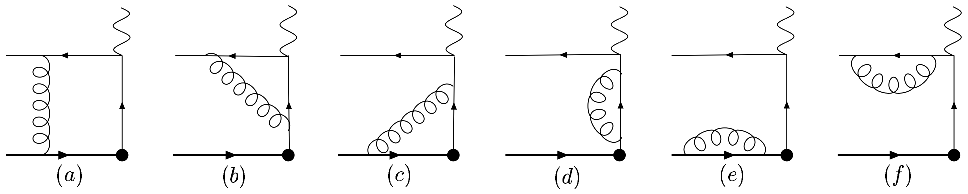

Figure 4: One-loop contribution to the matrix element. The thick line is for the b-quark, the black dot denotes the insertion of the operator.

with a partonic state of off-shell partons, one can have a $H^{(0)}$ which depends on $k_\perp$ [17]. But the amplitude T^μ with the state of off-shell partons is not gauge-invariant. In general it is not clear if the factorization with such a state can be made in a gauge invariant way.

At tree-level one can not determine the form of the soft factor $\tilde{S}$, because it is designed to subtract infrared divergences at higher orders of α_s. It should be a δ-function at tree level. At one-loop level with the partonic state, the factorization formula takes the form:

$$T_\mu^{(1)} \sim \phi_+^{(0)} \otimes \tilde{S}^{(0)} \otimes H^{(1)} + \phi_+^{(0)} \otimes \tilde{S}^{(1)} \otimes H^{(0)} + \phi_+^{(1)} \otimes \tilde{S}^{(0)} \otimes H^{(0)}, \tag{3.11}$$

the soft factor should be chosen so that all soft divergences of $T_\mu^{(1)}$ are contained in the second- and third term and $H^{(1)}$ is free from any soft divergence. The soft factor should also be chosen so that the factorization can be extended beyond one-loop level.

4. The soft factor and factorization at one-loop level

In this section we will perform TMD factorization at one-loop level and determine the operator form of the soft factor. The perturbative coefficient function will also be determined at one-loop level. Let us first consider the one-loop corrections to the amplitude $\mathcal{T}^\mu$. The corrections are from diagrams given in figure 4.

The contribution from figure 4a reads:

$$\begin{aligned} T^\mu|_{4a} &= iQ_q g_s^2 C_F \int \frac{d^4 l}{(2\pi)^4} \bar{v}(k_q)\gamma\cdot v \frac{\gamma\cdot(k_q+l)}{(k_q+l)^2 - m_q^2 + i\varepsilon}\gamma\cdot\epsilon^* \frac{\gamma\cdot(p-k_q-l)}{(p-k_q-l)^2 - m_q^2 + i\varepsilon} \\ &\quad \cdot\gamma^\mu(1-\gamma_5)\frac{1}{-v\cdot l + i\varepsilon}\frac{1}{l^2-\lambda^2+i\varepsilon}u(k_b) \\ &= iQ_q g_s^2 C_F \int \frac{d^4 l}{(2\pi)^4} \bar{v}(k_q)\gamma\cdot v \frac{\gamma\cdot(k_q+l)}{(k_q+l)^2 - m_q^2 + i\varepsilon}\gamma\cdot\epsilon^*\gamma\cdot p\gamma^\mu(1-\gamma_5)u(k_b) \\ &\quad \cdot\frac{1}{-2p\cdot(k_q+l)+i\varepsilon}\cdot\frac{1}{-v\cdot l+i\varepsilon}\cdot\frac{1}{l^2-\lambda^2+i\varepsilon} + \mathcal{O}(E_\gamma^{-2}). \end{aligned} \tag{4.1}$$

It is easy to find that this contribution up to a power correction is exactly represented by the contribution from $\phi_+|_{1a}$ to the third term in eq. (3.11). Therefore, this contribution and $\phi_+|_{1a}$ is irrelevant for the determination of $H^{(1)}$ and $\tilde{S}^{(1)}$. Also, the contributions from figure 4e and figure 4f to $T_\mu^{(1)}$ are reproduced by the contributions from $\phi_+|_{2a}$ and $\phi_+|_{2d}$ in the third term in eq. (3.11), respectively. The contributions from other diagrams to $T_\mu^{(1)}$ are:

$$\begin{aligned}
T^\mu|_{4b} &= T_\mu^{(0)}\cdot\frac{\alpha_s}{3\pi}\left[\ln\frac{\mu^2}{2k_q\cdot p}+2\ln\frac{2k_q\cdot p}{m_q^2}\right],\\
T_\mu|_{4c} &= T_\mu^{(0)}\cdot\frac{\alpha_s}{3\pi}\left[-\ln^2\left(\frac{2p\cdot k_q}{\zeta_v^2(k_q^+)^2}\right)+\ln\frac{\mu^2}{\zeta_v^2(k_q^+)^2}+2-\frac{4}{3}\pi^2\right],\\
T^\mu|_{4d} &= T_\mu^{(0)}\cdot\frac{\alpha_s}{3\pi}\left[\ln\frac{2k_q\cdot p}{\mu^2}-1\right].
\end{aligned}\tag{4.2}$$

Our $T^\mu|_{4b}$ agrees with that of [14], but is not in agreement with that in [13]. The other contributions are in agreement with [13]. In these contributions there are no infrared singularities. They have only a collinear singularity from figure 4b, represented by $\ln m_q$. The relevant contributions to $\phi_+^{(1)}\otimes\tilde S^{(0)}\otimes H^{(0)}$ by using $\tilde S^{(0)}$ are:

$$\begin{aligned}
W_b &= \int dk^+d^2k_\perp\frac{1}{k^+}\left[\phi_+|_{1b}(k^+,k_\perp)+\phi_+|_{2e}(k^+,k_\perp)+\phi_+|_{2f}(k^+,k_\perp)\right]/\bar v(k_q)\gamma^+\gamma_5u(k_b)\\
&= \frac{\alpha_s}{3\pi k_q^+}\left\{2+\ln\frac{\mu^2}{\zeta^2}-\frac{1}{2}\ln^2\frac{\lambda^2}{\zeta^2}+2\ln\frac{\zeta^2}{m_q^2}+\ln\frac{\mu^2}{\lambda^2}-\frac{1}{2}\pi^2-\int\frac{d^2k_\perp}{\pi}\frac{1}{\lambda^2+k_\perp^2}\ln\frac{\lambda^2+k_\perp^2}{\zeta^2}\right\},\\
W_c &= \int dk^+d^2k_\perp\frac{1}{k^+}\left[\phi_+|_{1c}(k^+,k_\perp)+\phi_+|_{2b}(k^+,k_\perp)+\phi_+|_{2c}(k^+,k_\perp)\right]/\bar v(k_q)\gamma^+\gamma_5u(k_b)\\
&= \frac{\alpha_s}{3\pi k_q^+}\left\{\ln\frac{\mu^2}{\lambda^2}+\ln\frac{\mu^2}{\lambda^2}\ln\frac{\zeta_v^2}{\zeta_u^2}-\frac{5}{6}\pi^2-\frac{1}{2}\ln^2\frac{\lambda^2}{\zeta_v^2(k_q^+)^2}-\int\frac{d^2k_\perp}{\pi}\frac{1}{\lambda^2+k_\perp^2}\ln\frac{\lambda^2+k_\perp^2}{\zeta^2}\right\},\\
W_d &= \int dk^+d^2k_\perp\frac{1}{k^+}\phi_+|_{1d}(k^+,k_\perp)/\bar v(k_q)\gamma^+\gamma_5u(k_b)=-\frac{2\alpha_s}{3\pi k_q^+}\int\frac{d^2k_\perp}{\pi}\frac{1}{\lambda^2+k_\perp^2}.
\end{aligned}\tag{4.3}$$

Comparing the above two equations, we find that the collinear singularity from figure 4b is reproduced by the contribution in W_b from figure 2f. But, there are many infrared singularities in $\phi_+^{(1)}\otimes\tilde S^{(0)}\otimes H^{(0)}\sim W_a+W_b+W_c+W_d+W_e$, where W_a is the contribution from figure 1a and W_e is the sum of contributions from figure 2a and figure 2d. There are even ultraviolet divergences. However these divergences are closely related to corresponding infrared singularities, as they stand. Once these infrared singularities are subtracted, one can expect that those ultraviolet divergences are subtracted too. As mentioned before, the contributions of W_a and W_e represent those to the T^μ from figure 4a, figure 4e and figure 4f. To complete the factorization one needs to find the soft factor so that all infrared singularities in $W_b+W_c+W_d$ and also the divergent integrals over $k_\perp$ are subtracted by the soft factor.

Clearly all these infrared singularities are from the TMD wave functions, i.e., from contributions from figure 1 and figure 2. By using the eikonal approximation one easily finds that these singularities can be reproduced by the expectation value of the product of gauge links:

$$\begin{aligned}
S_4(q^+,q_\perp) &= \int dz^-d^2z_\perp e^{iq^+z^--i\vec q_\perp\cdot\vec z_\perp}S_4(z^-,z_\perp),\\
S_4(z^-,z_\perp) &= \frac{1}{3}\mathrm{Tr}\langle 0|T\left[L_{\tilde u}^\dagger(z,-\infty)L_u^\dagger(\infty,z)L_u(\infty,0)L_v(0,-\infty)\right]|0\rangle|_{z^+=0},
\end{aligned}$$

$$
\begin{aligned}
L_{\tilde{u}}(z,-\infty) &= P\exp\left(-ig_s\int_{-\infty}^{0} d\lambda\tilde{u}\cdot G(\lambda\tilde{u}+z)\right),\\
L_{v}(z,-\infty) &= P\exp\left(-ig_s\int_{-\infty}^{0} d\lambda v\cdot G(\lambda v+z)\right),
\end{aligned}
\tag{4.4}
$$

where the direction in $L_{\tilde{u}}$ is chosen as $\tilde{u}^+ \gg \tilde{u}^-$. The gauge link $L_{\tilde{u}}^{\dagger}$ just simulates an anti-quark $\bar{q}$ in the initial state and L_v the b-quark in the initial state. If one only takes this product of the gauge links into account, one can expect that the quantity

$$
\frac{\phi_+(z^-,b,\zeta,\mu)}{S_4(z^-,b)}
\tag{4.5}
$$

is free from infrared singularities. This is checked at one-loop level. However, because part of contributions from $\phi_+^{(1)}$, which are from figure 1a, figure 2a and figure 2d, is already used up to subtract soft divergences in $T_\mu^{(1)}$, as discussed before, one can not expect that the soft factor $\tilde{S}$ can be formed with S_4 only. We will turn to this point later and concentrate at moment on the perturbative results for S_4.

At tree-level, the result is just a δ-function:

$$
S_4^{(0)}(q^+,q_\perp) = \delta(q^+)\delta^2(\vec{q}_\perp).
\tag{4.6}
$$

At one-loop level, there are contributions from diagrams which have a one-to-one correspondence to those diagrams given in figure 1 and figure 2, in which one only needs to replace the light-quark line with the double line of the gauge link $L_{\tilde{u}}^{\dagger}$. The corresponding contributions as a distribution of q^+ for the range $-k^+ < q^+ < \infty$ under the limits $u^+ \to 0$ and $\tilde{u}^- \to 0$ are:

$$
\begin{aligned}
S_4(q^+,\vec{q}_\perp)|_{1a} &= \frac{2\alpha_s}{3\pi^2}\left[\frac{\theta(q^+)}{q^+(q_\perp^2+\lambda^2+\zeta_v^2(q^+)^2)}+\frac{\theta(-q^+)}{q^+(q_\perp^2+\lambda^2+\zeta_{\tilde{u}}^2(q^+)^2)}\right]\\
&\quad + \text{“imaginary part”},\\
S_4(q^+,\vec{q}_\perp)|_{1b} &= -\frac{2\alpha_s}{3\pi^2}\left[\frac{1}{q_\perp^2+\lambda^2+\zeta_{\tilde{u}}^2(q^+)^2}\theta(-q^+)\left(\frac{1}{q^+}\right)_+ - \frac{1}{2}\delta(q^+)\frac{1}{q_\perp^2+\lambda^2}\ln\frac{\zeta^2}{q_\perp^2+\lambda^2}\right]\\
S_4(q^+,\vec{q}_\perp)|_{1c} &= \frac{2\alpha_s}{3\pi^2}\left[\left(\frac{\theta(q^+)}{q^+(q_\perp^2+\lambda^2+\zeta_v^2(q^+)^2)}\right)_+ + \frac{1}{2}\delta(q^+)\frac{1}{q_\perp^2+\lambda^2}\ln\frac{\zeta_u^2}{\zeta_v^2}\right],\\
S_4(q^+,\vec{q}_\perp)|_{1d} &= -\frac{2\alpha_s}{3\pi^2}\frac{\delta(q^+)}{q_\perp^2+\lambda^2},
\end{aligned}
\tag{4.7}
$$

and the contributions from the diagrams corresponding to those in figure 2 are:

$$
\begin{aligned}
S_4(q^+,\vec{q}_\perp)|_{2a} &= S_4(q^+,\vec{q}_\perp)|_{2b} = S_4(q^+,\vec{q}_\perp)|_{2d} = S_4(q^+,\vec{q}_\perp)|_{2e} = S_4^{(0)}(q^+,\vec{q}_\perp)\cdot\frac{\alpha_s}{3\pi}\ln\frac{\mu^2}{\lambda^2},\\
S_4(q^+,\vec{q}_\perp)|_{2c} &= -S_4^{(0)}(q^+,\vec{q}_\perp)\frac{\alpha_s}{3\pi}\ln\frac{\mu^2}{\lambda^2}\ln\frac{\zeta_u^2}{\zeta_v^2},\\
S_4(q^+,\vec{q}_\perp)|_{2f} &= -S_4^{(0)}(q^+,\vec{q}_\perp)\frac{\alpha_s}{3\pi}\ln\frac{\mu^2}{\lambda^2}\ln\frac{\zeta_u^2}{\zeta_{\tilde{u}}^2},
\end{aligned}
\tag{4.8}
$$

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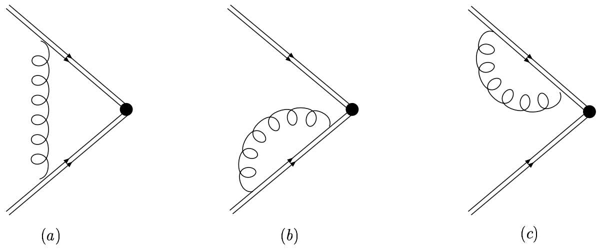

Figure 5: One-loop contribution to S_2. The double lines represent the two gauge links. One is for L_v, the other one is for $L^\dagger_{\tilde u}$.

with $\zeta^2_{\tilde u} = 2\tilde u^-/\tilde u^+$. If we identify the soft factor $\tilde S(z^-, \vec b)$ as $S_4^{-1}(z^-, \vec b)$, their contributions to $\phi^{(0)}_+ \otimes \tilde S^{(1)} \otimes H^{(0)}$ can be grouped similarly as those to $\phi^{(1)}_+ \otimes \tilde S^{(0)} \otimes H^{(0)}$. They are:

$$\begin{aligned}
U_a &= -\int dl^+ d^2 l_\perp \frac{1}{k_q^+ + l^+} S_4|_{1a}(l^+, l_\perp) \\
&= \frac{\alpha_s}{3\pi k_q^+}\left\{\frac{1}{2}\ln^2\frac{\zeta_v^2 (k_q^+)^2}{\lambda^2} + \frac{5\pi^2}{6} - \frac{1}{2}\ln^2\frac{\zeta_{\tilde u}^2 (k_q^+)^2}{\lambda^2} + \Delta\right\} + \text{``imaginary part''}, \\
U_b &= -\int dl^+ d^2 l_\perp \frac{1}{k_q^+ + l^+}\left[S_4|_{1b} + S_4|_{2e} + S_4|_{2f}\right](l^+, l_\perp) \\
&= \frac{\alpha_s}{3\pi k_q^+}\left\{-\Delta + \frac{1}{2}\ln^2\frac{\lambda^2}{\zeta_{\tilde u}^2 (k_q^+)^2} - \ln\frac{\mu^2}{\lambda^2} - \ln\frac{\mu^2}{\lambda^2}\ln\frac{\zeta_{\tilde u}^2}{\zeta_u^2} + \int\frac{d^2k_\perp}{\pi}\frac{1}{k_\perp^2+\lambda^2}\ln\frac{k_\perp^2+\lambda^2}{\zeta^2}\right\}, \\
U_c &= -\int dl^+ d^2 l_\perp \frac{1}{k_q^+ + l^+}\left[S_4|_{1c} + S_4|_{2b} + S_4|_{2c}\right](l^+, l_\perp) \\
&= \frac{\alpha_s}{3\pi k_q^+}\left\{\frac{5}{6}\pi^2 + \frac{1}{2}\ln^2\frac{\lambda^2}{\zeta_v^2 (k_q^+)^2} - \ln\frac{\mu^2}{\lambda^2} - \ln\frac{\mu^2}{\lambda^2}\ln\frac{\zeta_v^2}{\zeta_u^2} + \int\frac{d^2k_\perp}{\pi}\frac{1}{k_\perp^2+\lambda^2}\ln\frac{k_\perp^2+\lambda^2}{\zeta^2}\right\}, \\
U_d &= -\int dl^+ d^2 l_\perp \frac{1}{k_q^+ + l^+} S_4|_{1d}(l^+, l_\perp) = \frac{2\alpha_s}{3\pi k_q^+}\int\frac{d^2k_\perp}{\pi}\frac{1}{k_\perp^2+\lambda^2}, \\
U_e &= -\int dl^+ d^2 l_\perp \frac{1}{k_q^+ + l^+}\left[S_4|_{2a} + S_4|_{2d}\right](l^+, l_\perp) = -\frac{2\alpha_s}{3\pi}\ln\frac{\mu^2}{\lambda^2},
\end{aligned} \tag{4.9}$$

where Δ is a divergent quantity:

$$\Delta = \lim_{k^+\to 0}\ln\frac{k_q^+}{k^+}\int\frac{d^2k_\perp}{\pi}\frac{2}{\zeta_{\tilde u}^2 (k_q^+)^2 + k_\perp^2}, \tag{4.10}$$

which will be cancelled in $U_a + U_b$. Comparing the sum $U_a + U_b + U_c + U_d + U_e$ with $W_b + W_c + W_d$, we note first that the divergent integrals over $k_\perp$ in W_b, W_c and W_d are completely subtracted by those in U_b, U_c and U_d, respectively. Also the infrared singularities with $\ln\lambda$ in W_b, W_c and W_d are completely subtracted by those in U_b, U_c and U_d, respectively. The remaining infrared singularities are only from U_a and U_e.

These remaining singularities can be reproduced by the product of the gauge links:

$$S_2 = \frac{1}{3}\mathrm{Tr}\langle 0|T\left[L^\dagger_{\tilde u}(0,-\infty)L_v(0,-\infty)\right]|0\rangle. \tag{4.11}$$

At leading order $S_2^{(0)} = 1$. At one-loop level, the contributions are from the diagrams given in figure 5.

$$\begin{aligned} S_2|_{5a} &= -\frac{\alpha_s}{3\pi}\ln\frac{\mu^2}{\lambda^2}\ln\frac{\zeta_v^2}{\zeta_{\tilde{u}}^2} + \text{“imaginary part”}, \\ S_2|_{5b} &= S_2|_{5c} = \frac{\alpha_s}{3\pi}\ln\frac{\mu^2}{\lambda^2}. \end{aligned} \tag{4.12}$$

Now we turn to the imaginary or absorptive part. In the amplitude T^μ it has an absorptive part from the box diagram figure 4a and its contribution is already contained in the contribution from the TMD wave function in figure 1a. The remaining parts T^μ can not have an absorptive part. Therefore, one should eliminate possible absorptive part in the soft factor. At one-loop level, the imaginary part from S_4 is the same as that from S_2. But this is from perturbative theory. To eliminate the absorptive part one can simply take the real parts of those products of gauge links. Therefore we determine the soft factor as:

$$\begin{aligned} \tilde{S}(z^-,\vec{b},\zeta_u,\mu) &= \frac{\mathrm{Re}\left[S_2(\zeta_{\tilde{u}},\mu)\right]}{\mathrm{Re}\left[S_4(z^-,\vec{b},\zeta_u,\zeta_{\tilde{u}},\mu)\right]}, \\ \tilde{S}(k^+,k_\perp,\zeta,\mu) &= \frac{1}{(2\pi)^3}\int dz^- d^2b e^{ik^+z^- - i\vec{k}_\perp\cdot\vec{b}}\tilde{S}(z^-,\vec{b},\zeta_u,\mu). \end{aligned} \tag{4.13}$$

It should be noted that S_2 and S_4 depend on $\zeta_{\tilde{u}}$, but the soft factor as the ratio of them does not depend on $\zeta_{\tilde{u}}$. With the defined soft factor, the form factors can be factorized as in eq. (3.9). They take a compact form in the b-space:

$$F_V = F_A = \frac{iQ_q}{2v\cdot p}\lim_{b\to 0}\int dk^+ dl^+ \phi_+(k^+,b,\zeta,\mu)\tilde{S}(l^+,b,\zeta_u,\mu)\theta(k^+ + l^+)H(k^+ + l^+,\zeta_u,\mu). \tag{4.14}$$

The limit $b \to 0$ should be taken after the integrations. With the results presented before, we determine the perturbative coefficient function H as:

$$\begin{aligned} H(k^+,\zeta_u,\mu) = \frac{1}{k^+}\Bigg\{1 + \frac{2\alpha_s(\mu)}{3\pi}\Bigg[&-\frac{1}{2}\ln^2\frac{2k\cdot p}{\zeta_v^2(k^+)^2} + \frac{1}{4}\ln\frac{\zeta_u^2}{\zeta_v^2}\left(\ln\frac{\zeta_u^2(k^+)^2}{\mu^2} + \ln\frac{\zeta_v^2(k^+)^2}{\mu^2}\right) \\ &+\frac{1}{2}\ln\frac{2k\cdot p}{\zeta_v^2(k^+)^2} + \frac{1}{2}\ln\frac{2k\cdot p}{\zeta_u^2(k^+)^2} - \frac{1}{2} - \frac{5\pi^2}{6}\Bigg]\Bigg\} + \mathcal{O}(\alpha_s^2), \end{aligned} \tag{4.15}$$

which is free from any soft divergence. All soft singularities are cancelled on a diagram-by-diagram basis. The cancellation on a diagram-by-diagram basis is important for extending the factorization beyond one-loop level. General arguments for the factorization at any loop can be given by performing an analysis of relevant reduced diagrams and infrared power-counting. The perturbative coefficient function H here does not contain the double log $\ln^2\mu^2$ in contrast with that in the collinear factorization [14–16], instead of $\ln^2\mu^2$ it contains $\ln^2\zeta_u^2$ and other log terms. All of those log terms need to be resummed if they can be large.

It should be noted that for the case studied here one may redefine the TMD light-cone wave function by including the soft factor as $\phi'_+ = \phi_+ \otimes \tilde{S}$, so that the form factors take the form $\phi'_+ \otimes H$. Then our results look similar to those in the collinear factorization. However, it is not clear if the same can be done for other processes, because they have not been studied yet. Therefore, we leave the soft factor there explicitly.

5. Relation between two factorizations

In the section we show that the result of the collinear factorization for the decay can be obtained from that of TMD factorization, which is given in the last section. Hence, a simple relation between two factorizations is found for the decay.

In collinear factorization, the transverse momenta of partons are integrated out and the collinear light-cone wave function can be defined as [19]:

$$\Phi_+(k^+,\mu) = \int \frac{dz^-}{2\pi} e^{ik^+z^-} \langle 0|\bar{q}(z^-n)L_n^\dagger(\infty, z^-n)\gamma^+\gamma_5 L_n(\infty,0)h(0)|\bar{B}(v)\rangle, \tag{5.1}$$

with the gauge link L_n defined with the light-cone vector $n^\mu = (0,1,0,0)$:

$$L_n(\infty,z) = P\exp\left(-ig_s\int_0^\infty d\lambda n\cdot G(\lambda n+z)\right). \tag{5.2}$$

By taking the same partonic state as given in section 2., the wave function can be calculated with perturbative QCD. The result can be found in [10]. With this result and that in the last section, one can easily derive the result in the collinear factorization:

$$F_V = F_A = \frac{iQ_q}{2v\cdot p}\int dk^+\Phi_+(k^+,\mu)H_c(k^+,\mu), \tag{5.3}$$

where H_c is the perturbative coefficient function and is given by:

$$\begin{aligned} H_c(k^+,\mu) \;=\; & \frac{1}{k^+}\left\{1+\frac{\alpha_s}{3\pi}\left[\frac{1}{2}\ln^2\frac{\mu^2}{\zeta_v^2(k^+)^2}+\ln\frac{2k\cdot p}{\mu^2}+\ln\frac{2k\cdot p}{\zeta_v^2(k^+)^2}-\ln^2\frac{2k\cdot p}{\zeta_v^2(k^+)^2}\right.\right. \\ & \left.\left. -3-\frac{7\pi^2}{12}\right]\right\}+\mathcal{O}(\alpha_s^2), \end{aligned} \tag{5.4}$$

where the logarithmic terms agree with those in [14−16]. The difference in constant terms is caused by that we used HQET for T^μ, while full QCD was used to calculate it in [14−16].

The TMD light-cone wave function has a factorized relation to Φ_+ in b-space [10]. It reads:

$$\phi_+(k^+,b,\zeta,\mu) = \int_0^\infty dq^+ C_B(k^+,q^+,b,\zeta,\mu)\Phi_+(q^+,\mu)+\mathcal{O}(b), \tag{5.5}$$

where the function C_B can be determined by perturbative theory and is free from any soft divergence. At leading order of α_s the function $C_B(k^+,q^+,b,\zeta,\mu)$ is $\delta(k^+-q^+)$. The result of C_B at one-loop level can be found in [10]. It should be noted that from the results in section 2. the TMD light-cone wave function $\phi_+(k^+,k_\perp,\zeta)$ at one loop order in the momentum space contains various infrared divergences. Some of them are proportional to the tree-level result, i.e., to $\delta^2(\vec{q}_\perp)$, some of them take a form like $1/(q_\perp^2+\lambda^2)$. These singularities do not cancel if $\vec{q}_\perp$ goes to zero. But, when we transform $\phi_+(k^+,k_\perp,\zeta)$ into

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the b-space, i.e., when we integrate over $k_\perp$, some of these singularities are cancelled, the remaining singularities are just the same as those in Φ_+. Therefore C_B is free from any soft divergences. The same also happens to the soft factor $\tilde{S}$ with the difference that the infrared singularities are completely cancelled, if we transform it into the b-space, or we integrate over the transverse momentum. The soft factor $\tilde{S}$ in b-space reads:

$$\begin{aligned}\tilde{S}(q^+,\vec{b},\zeta_u,\mu) &= \delta(q^+) + \frac{4\alpha_s}{3\pi}\theta(q^+)\left(\frac{\ln(\tilde{b}^2\zeta_v^2(q^+)^2)}{q^+}\right)_+ \\ &\quad + \frac{2\alpha_s}{3\pi}\delta(q^+)\left[\ln(\tilde{b}^2\mu^2)\left(\ln\frac{\zeta_u^2}{\zeta_v^2}-1\right)+\frac{\pi^2}{6}+\frac{1}{2}\ln^2(\tilde{b}^2(P^+)^2\zeta_v^2)\right]+\mathcal{O}(b^2),\end{aligned} \tag{5.6}$$

with $\tilde{b} = be^\gamma/2$. Here $\tilde{S}(q^+,\vec{b},\zeta_u,\mu)$ should be taken as a distribution for $q^+ < P^+$. The heavy quark limit implies $P^+ \to \infty$. As discussed before and in [10], we should take finite P^+ in the calculation and take the limit in the final result. The same also applies for eq. (5.5), where the upper bound of q^+ should be taken as P^+. With these results our factorization formula can be re-written as:

$$\begin{aligned}F_V = F_A = \frac{iQ_q}{2v\cdot p}\lim_{b\to 0}\lim_{P^+\to\infty}\int_0^{P^+} dq^+\Phi_+(q^+,\mu)\Big\{&H^{(0)}(q^+,\zeta_u,\mu)+H^{(1)}(q^+,\zeta_u,\mu)\\ &+\int_0^{P^+} dk^+ C_B^{(1)}(k^+,q^+,\zeta,\mu)H^{(0)}(l^+,\zeta_u,\mu)\\ &+\int_{-q^+}^{P^+} dl^+\tilde{S}^{(1)}(l^+,b,\zeta_u,\mu)H^{(0)}(l^++q^+,\zeta_u,\mu)\Big\}+\mathcal{O}(\alpha_s^2).\end{aligned} \tag{5.7}$$

With our results of $C_B^{(1)}$, $\tilde{S}^{(1)}$ and $H^{(1)}$ we reproduce H_c in eq. (5.4). Therefore, the two factorizations with fixed orders of perturbative theory are equivalent.

6. Resummation of large logarithms

In general, one expects that the most important k^+-region of $\phi^+(k^+,k_\perp,\zeta)$ for a convolution with the wave function like eq. (4.14) will be around $k^+ \sim \Lambda_{QCD}$. Also the important region of the soft factor $\tilde{S}(l^+,l_\perp,\zeta_u)$ is with small l^+, i.e., $l^+ \sim \Lambda_{QCD}$. This results in that $H^{(1)}(k^+ + l^+)$ will contain large single logarithms and large double logarithms and it spoils the perturbative expansion of H. Those large logarithms should be resummed for a reliable prediction.

In the collinear factorization in [14–16], the resummation can be done by introducing a jet factor in the frame work of the soft collinear effective theory [22], or a jet factor in the full QCD [18]. Similarly, we can also introduce a jet factor in our factorization for the resummation. However, as we have seen before, our TMD light-cone wave function and soft factor depend on the parameter ζ_u. This dependence can be used to resum those large logarithms. Before showing this, let us first study the evolution of the soft factor.

The evolution with the renormalization μ and with the parameter ζ_u reads:

$$\mu\frac{\partial}{\partial\mu}\tilde{S}(k^+,\vec{k}_\perp,\zeta_u,\mu) = \frac{4\alpha_s}{3\pi}\left[\ln\frac{\zeta_u^2}{\zeta_v^2}-1\right]\tilde{S}(k^+,\vec{k}_\perp,\zeta_u,\mu)+\mathcal{O}(\alpha_s^2),$$

$$
\begin{aligned}
\zeta_u \frac{\partial}{\partial \zeta_u} \tilde{S}(k^+, b, \zeta_u, \mu) &= \frac{4\alpha_s}{3\pi} \left[2\gamma - \ln 4 + \ln b^2\mu^2 \right] \tilde{S}(k^+, b, \zeta_u, \mu) + \mathcal{O}(\alpha_s^2), \\
\mu \frac{\partial}{\partial \mu} \phi_+(k^+, k_\perp, \zeta, \mu) &= \frac{\alpha_s}{\pi} \left[1 + \frac{2}{3} \left(2 - \ln \frac{\zeta_u^2}{\zeta_v^2} \right) \right] \phi_+(k^+, k_\perp, \zeta, \mu) + \mathcal{O}(\alpha_s^2), \\
\zeta \frac{\partial}{\partial \zeta} \phi_+(k^+, b, \zeta, \mu) &= \left[-\frac{4\alpha_s}{3\pi} \ln \frac{\zeta^2 b^2 e^{2\gamma - 1}}{4} - \frac{2\alpha_s}{3\pi} \ln \frac{\mu^2 e}{\zeta^2} \right] \phi_+(k^+, b, \zeta, \mu) + \mathcal{O}(\alpha_s^2), \quad (6.1)
\end{aligned}
$$

where we also include the evolutions of the wave function for completeness. With these equations, one can show that the form factors in eq. (3.9) or eq. (4.14) are independent of ζ_u^2, as expected. Also, their μ-dependence is compensated by the μ-dependence of $f(\mu)$ in eq. (3.3) so that the matrix element $\langle \gamma(\epsilon^*, p) | \bar{q}\gamma^\mu(1-\gamma_5) b | \bar{B}(v) \rangle$ does not depend on μ.

To resum the large log terms, we first take an initial value $\zeta_u = \zeta_{u0}$ in eq. (4.14) so that there are no large log terms introduced by ζ_{u0} in the wave function and the soft factor. Then there will be large log terms with ζ_{u0} in the coefficient function H, which can be re-expressed with little algebra as:

$$
\begin{aligned}
H(q^+, \zeta_{u0}, \mu) = \frac{1}{q^+} \Bigg\{ 1 + \frac{2\alpha_s}{3\pi} \Bigg[& \frac{1}{4} \ln^2 \left(\frac{\zeta_{u0}^2 (q^+)^2}{e\mu^2} \right) - \frac{3}{2} \ln^2 \left(\frac{\mu}{q^+} \left(\frac{\zeta_v^4 \mu}{2p^-} \right)^{-\frac{1}{3}} \right) \\
& - \frac{1}{3} \left(\ln \frac{2p\cdot v}{\mu} - \frac{3}{2} \right)^2 - \frac{5\pi^2}{6} \Bigg] \Bigg\}, \qquad (6.2)
\end{aligned}
$$

where $q^+ = k^+ + l^+$. For small k^+ and l^+ there are large log terms in the first line. These terms can be resummed by using the ζ_u-evolution of H:

$$
\zeta_u \frac{\partial}{\partial \zeta_u} H(q^+, \zeta_u, \mu) = \frac{2\alpha_s}{3\pi} \left[\ln \frac{\zeta_u^2 (q^+)^2}{\mu^2} - 1 \right] H(q^+, \zeta_u, \mu) + \mathcal{O}(\alpha_s^2). \qquad (6.3)
$$

Solving this equation we have:

$$
\begin{aligned}
H(q^+, \zeta_{u0}, \mu) = \exp & \left\{ + \frac{\alpha_s(\mu)}{6\pi} \left[\ln^2 \left(\frac{\mu^2 e}{\zeta_{u0}^2 (q^+)^2} \right) - \frac{2}{3} \ln^2 \left(\frac{\zeta_v^4 (q^+)^4}{2p^-(q^+)\mu^2} \right) \right] \right\} \\
& \cdot \frac{1}{q^+} \left\{ 1 + \frac{2\alpha_s(\mu)}{3\pi} \left[-\frac{1}{3} \left(\ln \frac{2p\cdot v}{\mu} - \frac{3}{2} \right)^2 - \frac{5\pi^2}{6} \right] \right\}. \qquad (6.4)
\end{aligned}
$$

All large log terms due to small +-momenta are now resummed in the exponential. To eliminate the large log terms in the second line of the above equation, we can set $\mu = \mu_H$ with a large μ_H so that the $\ln(2p \cdot v/\mu_H)$ is a number of order 1. Then there are large log terms due to large μ_H in the wave function and the soft factor. With the evolution equations in eq. (6.1), we can evolute them to lower scales as $\mu = \mu_0$. Finally we have for the form factors:

$$
\begin{aligned}
F_V = F_A = & \frac{iQ_q}{2v\cdot p} \lim_{b\to 0} \int dk^+ dl^+ \phi_+(k^+, b, k^+\zeta_{u0}, \mu_0) \tilde{S}(l^+, b, \zeta_{u0}, \mu_0) \theta(k^+ + l^+) \\
& \cdot e^{S_F(k^+ + l^+)} \cdot \frac{1}{k^+ + l^+} \left\{ 1 + \frac{2\alpha_s(\mu_H)}{3\pi} \left[-\frac{1}{3} \left(\ln \frac{2p\cdot v}{\mu_H} - \frac{3}{2} \right)^2 - \frac{5\pi^2}{6} \right] \right\}, \quad (6.5)
\end{aligned}
$$

with

$$S_F(q^+) = \frac{\alpha_s(\mu_H)}{6\pi}\left[\ln^2\left(\frac{\mu_H^2 e}{\zeta_{u0}^2(q^+)^2}\right) - \frac{2}{3}\ln^2\left(\frac{\zeta_v^4(q^+)^4}{2p^-(q^+)\mu_H^2}\right)\right] + \int_{\mu_0}^{\mu_H}\frac{d\mu}{\mu}\frac{\alpha_s(\mu)}{\pi}\left(1+\frac{2}{3}\ln\frac{\zeta_{u0}^2}{\zeta_v^2}\right). \tag{6.6}$$

In the above all large logs are resummed in the factor e^{S_F}. The initial value μ_0 should be taken where perturbative QCD is still applicable. One may take $\mu_0 = 1 \sim 2$GeV. For ζ_{u0}, with our definitions of the TMD light-cone wave function and the soft factor, we should have $\zeta_{u0} \gg 1$, although the physics here, i.e., the form factors, does not depend on ζ_{u0}. However, one should not take a too large ζ_{u0} to avoid large log terms in the wave function and the soft factor. A detailed study of a reasonable choice of ζ_{u0} and μ_0 is needed when one uses the factorization results for phenomenological applications.

For the resummed results one can also use the relation in eq. (5.5) and the result in eq. (5.6) to express them in term of the usual light-cone wave function Φ_+, as in the last section. Then instead of the integrand $C_B^{(1)} \otimes H^{(0)}$ in eq. (5.7) we have a complicated integrand $C_B^{(1)} \otimes H^{(0)} \otimes e^{S_F}$. Unfortunately, we are unable to calculate the integral analytically. The same also applies to the term corresponding to the term in the third line of eq. (5.7). Here, we only remind that our resummed form factors can be expressed as a convolution of Φ_+ with other functions.

7. Conclusion and outlook

As mentioned in the introduction, there are two approaches for exclusive B-decays. The two approaches are not only different in their formulations but also in some predictions in comparison with experiment. This leads to controversial discussions, e.g., see [11, 14, 23, 24]. Since two approaches are from one fundamental theory-QCD, there must be some relations between them. With a consistent definition of TMD light-cone wave functions these relations can be explored and predictions for exclusive B-decays from the two approaches may be unified. For this purpose, a first step is to define the TMD light-cone wave function consistently and to obtain relations between the TMD light-cone wave function and the usual light-cone wave function in the collinear factorization. This has been done in our previous work [10].

In this paper, we have shown that with the consistent definition of the TMD light-cone wave function the TMD factorization for the radiative leptonic B-decay can be performed consistently at one-loop level. In this factorization, beside the wave function as a nonperturbative object, another nonperturbative object, which is the soft factor, must be introduced, so that the perturbative coefficient function is free from any soft divergence. The results are given in eq. (4.14) and eq. (4.15). An extension of our factorization beyond one-loop level is possible.

The TMD light-cone wave function defined in [10] does not only depend on +- and transverse components of parton momentum, but also depends on the parameter ζ_u which regularizes the light-cone singularity. This ζ_u-dependence can be used to resum large logarithms in the perturbative coefficient function, as we have shown in section 6.

For the decay studied here, we can show that the result of collinear factorization can be derived from that of our TMD factorization. Hence the two factorizations are related to each other. This simple relation is to be expected because there is no hadrons, or partons in the final state. In other cases, the relation can be expected to be complicated.

Having shown that TMD factorization works in the simple case, we are ready to explore how TMD factorization works in other complicated cases and how it is related to the collinear factorization in these cases. Works for this are in progress.

Acknowledgments

This work is supported by National Nature Science Foundation of P.R. China.

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